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The Variation of Financial Arbitrage via the Use of an Information Wave Function

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Abstract We define an 'information wave function', $\Psi(q)$. We underline the role of risk-neutral probabilities in financial non-arbitrage. We argue how a change in the probabilities based on $\Psi(q)$ can induce arbitrage.

Keywords Information wave function · Risk-neutral probabilities · Arbitrage

1 Introduction

The concept of arbitrage is an important concept in financial economics. Many financial institutions make sizable profits on the basis of 'arbitrage'. What is arbitrage? The absence of arbitrage is equivalent to saying, as per Higham [11], that "(t)here is never an opportunity to make a risk-free profit that gives a greater return than that provided by the interest from a bank deposit." We can think of the bank deposit interest as being equal to the risk-free rate of return. Why a risk-free rate of return? The return is risk-free (and thus low) because the risk of losing (or receiving less back than what was put in on the bank deposit originally) is very low. We differentiate the risk-free rate of return from the risky rate of return (or the return in excess of the risk-free rate of return) when we consider risky investments (i.e. investments which may give us back less money (or much more money) than what was put in originally). So any *risk-free* profit which has a higher return than this risk-free rate of return is an arbitrage profit. The key requirement for obtaining an arbitrage profit is that the profit is risk-free (i.e. no risk has been taken to realize such profit).

Assets in financial economics are theoretically priced via a theorem, also known as the non-arbitrage theorem. This theorem allows for the generation of benchmark prices which exist under no arbitrage. In this paper we specify this theorem.

The contribution of this paper consists in seeing how information can be modelled, via the use of a so called information wave function, as a trigger for generating arbitrage.

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In Sect. 2 we define and discuss the non-arbitrage theorem. In Sect. 3, we discuss the so called information wave function and we also query why this function is to be a quantum mechanical wave function. In Sect. 4 we show how the information wave function can be used to induce arbitrage. In Sect. 5, we expand on how we can connect the non-arbitrage theorem with Bohmian mechanics.

2 The Non-Arbitrage Theorem

In order to define the non-arbitrage theorem we need the concept of risk-neutral probability. Those probabilities can be defined as follows [18].

Definition 2.1 Consider a probability measure Q on $\Omega = \{\omega_1, \omega_2, \dots, \omega_K\}$, where Ω is a finite sample space containing states of the world, ω_i, $i = 1, 2, \dots, K$. We note that a state of the world is for instance the state of the stock market: 'bullish' or 'bearish'. The probability measure Q on Ω is risk-neutral if (a) $\forall \omega_i \in \Omega : Q(\omega_i) > 0$ and (b) $E^Q[\Delta S_j] = 0$, $j = 1, 2, \dots, N$ (N indicates the number of securities), which is equivalent to $E^Q[S_j(1)] = S_j(0)$, where $E^Q[\Delta S_j]$ indicates the expectation operator on the random variable ΔS_j under the probability measure Q. ΔS_j indicates the discrete change in price of security S_j over two time points (i.e. respectively $S_j(1)$ versus $S_j(0)$). At time $t_0 < t_1$, the value of the state of the world at time t_1 is unknown.

Let us give an example of a risk-neutral probability.

Example 2.1 Consider $3Q(\omega_1) + 6Q(\omega_2) = 5$. Can we find $Q(\omega_1)$ and $Q(\omega_2) > 0$ so that $E^Q[S_1(1)] = S_1(0)$? Clearly, for $Q(\omega_1) = 1/3$ and $Q(\omega_2) = 2/3$, we obtain that $E^Q[S_1(1)] = 3(1/3) + 6(2/3) = 5 = S_1(0)$. Since Q is a probability measure we must have that $Q(\omega_1) + Q(\omega_2) = 1$ which is thus verified here.

The next theorem, also known under the name of the 'fundamental theorem of asset pricing' or also as the 'non-arbitrage theorem', gives the exact conditions under which we can guarantee no arbitrage. The theorem was originally formulated by Harrison and Kreps [7]. In this paper, we follow Etheridge [6] for the formulation of the theorem. We have adapted the notation slightly from Etheridge [6].

Theorem 2.1 *Assume there are N tradable assets* (*some assets may be risky and some not*) *and their prices*, *at time t_0 are given by $\overrightarrow{p_0}=(p_0^1, p_0^2, \dots, p_0^N)$. Assume there exists a K* (*where K indicates the K states of the world*) *dimensional state price vector $\overrightarrow{\Phi} = (\Phi_1, \Phi_2, \dots, \Phi_K)$ which is strictly positive in all coordinates. Consider the following model*:

$$\begin{pmatrix} p_0^1 \\ p_0^2 \\ \vdots \\ p_0^N \end{pmatrix} = \Phi_1 \begin{pmatrix} D_{11} \\ D_{21} \\ \vdots \\ D_{N1} \end{pmatrix} + \Phi_2 \begin{pmatrix} D_{12} \\ D_{22} \\ \vdots \\ D_{N2} \end{pmatrix} + \cdots + \Phi_K \begin{pmatrix} D_{1K} \\ D_{2K} \\ \vdots \\ D_{NK} \end{pmatrix},$$

where each N dimensional vector $\overrightarrow{D_1}, \dots, \overrightarrow{D_K}$ is the security price vector at time t_1, *if the market is*, *respectively*, *in state $1, \dots, K$. For the market model described here there is no arbitrage if and only if there is a state price vector.*

Etheridge [6] indicates that the proof of this theorem is an application of the Hahn-Banach Separation theorem. Please see [6] or [5] for the proof. Generalizations of this theorem exist. See for instance [13].

It is important to stress how the risk-neutral probabilities can be constructed out of Theorem 2.1. As per Etheridge [6] one can define the vector, $\overrightarrow{\Phi_{\text{prob}}} = (\frac{\Phi_1}{\Phi_0}, \frac{\Phi_2}{\Phi_0}, \frac{\Phi_3}{\Phi_0}, \ldots, \frac{\Phi_K}{\Phi_0})$, where each coordinate is a probability and $\Phi_0 = \exp(-rT)$ is the discount rate (continuously discounted) at the risk-free rate of return, r, and T is time. What is beautiful is that under $\overrightarrow{\Phi_{\text{prob}}}$ it is easy to show that the rate of return of a risky security is the risk-free rate of return! This is a very useful result since all participants in the economy can agree upon the level of the risk free-rate of return. On the contrary if the rate of return were to be the risky rate of return then such return would depend on each individuals' preferences for risk. In that case there would not be unanimous agreement on what the level of the return should be. As per Etheridge [6] $E^{\overrightarrow{\Phi_{\text{prob}}}}(p_T^j) = \sum_{i=1}^K D_{ij} \frac{\Phi_i}{\Phi_0}$, where $i = 1, \ldots, K$ and $j = 1, \ldots, N$ and where $E^{\overrightarrow{\Phi_{\text{prob}}}}$ is the expectation operator under the risk-neutral probability measure. This result leads us into the application of martingales in economics. We do not expand on it here.

3 The Information Wave Function and Quantum Mechanics

So far we have considered the non-arbitrage theorem. The contribution of this paper does of course not consist in re-citing this theorem and explaining some of its intricacies. Our contribution in this paper consists in showing how the information wave function, which we are about to discuss, can induce, in a very explicit way, arbitrage. Let us first define the quantum mechanical wave function (in polar form) as:

$$\Psi(q, t) \equiv R(q, t) \exp(iA(q, t))/\hbar; \tag{1}$$

where $R(q, t)$ is the amplitude function; $A(q, t)$ is the phase function; $\hbar$ is the Planck constant and i is a complex number. We note that t is time. Moreover, q is position. We note that q could be an n-dimensional vector. For instance, using the notation of the former section, we could for instance set $n = N$, where the vector q would then refer to the prices of N risky and riskless assets. This vector q would belong to, what Choustova [4] calls, a price configuration space, $Q = \mathbb{R}^n$. A space of price changes, $V = \mathbb{R}^n$, as in Choustova [4], could also be introduced. This leads to the existence of a price phase space: $Q \times V = \mathbb{R}^{2n}$. Another interpretation for q will be suggested, after we have covered Proposition 4.1 (please see Sect. 4). We will then interpret q as the price of information. This price could then refer to for instance the price of proprietary information. An n-dimensional vector q would then refer to the prices of n proprietary information data-sources. Examples of such data sources could be Bloomberg or Reuters. However, one may consider other data sources such as the data sources that research departments of banks construct to help in tracing arbitrage opportunities on specific financial products.

Three immediate questions now arise:

1. Why, in the macro-scopic-financial economics context of this paper, do we need a quantum mechanical wave function?
2. Why is this wave function interpreted as an information wave function?
3. What interpretation can we give to $\hbar$ in a macro-scopic-financial economics context?

We believe that the first two questions can begin to be answered by putting forward the argument that Bohmian mechanics (please see below in this section) is an interpretation of quantum mechanics which is particularly useful in the financial economics context we are presenting in this paper. In Choustova [4], the space $L^2(\mathbb{R}^n)$ of square integrable functions $\Psi : Q = \mathbb{R}^n \to \mathbb{C}$ is considered. As we have indicated above, we could consider a price configuration space of n-dimensional price vectors. For the purposes of this paper however, let us consider the simple case of $n = 1$. We claim that Ψ belongs to the space $L^2(\mathbb{R}, dM)$ of square integrable functions with respect to some measure M on $\mathbb{R}$ such that:

$$\|\Psi\|^2 = \int_{\mathbb{R}} |\Psi(q)|^2 dM(q) < \infty. \tag{2}$$

What is of high interest is that the measure M can describe what Choustova [4] calls "the classical random fluctuations." The information effects are then described by Ψ which is now the information wave function. The idea of using Ψ as an information wave function originates from the work of Bohm and Hiley [3], in which the authors likened the pilot wave function to a radio wave steering a ship on automatic pilot. The pilot wave function concept was developed by Bohm and Hiley [1–3]. Please see also [12] for an excellent account of Bohmian mechanics. Important work on using Bohmian mechanics in an economics context was first started by Khrennikov [14–16] and Choustova [4]. Haven [8, 10] attempts to show how their approach can be used in specific asset pricing contexts. The essential idea of Bohmian mechanics, we want to follow in this paper is that the wave function steers the particle. This idea was in some sense already contained in the work of de Broglie who attributed two roles to the wave function (p. 16 in [12]): "not only does it determine the likely location of a particle it also influences the location by exerting a force on the orbit."

In order to connect Bohmian mechanics with the non-arbitrage theorem, we first need to formulate the main proposition (Proposition 4.1) of this paper. We formulate this proposition in the next section. Thereafter, in the last section of this paper, we will argue how we can connect the pilot wave concept of Bohmian mechanics with the non-arbitrage theorem.

Finally, before closing this section, we need to attempt to answer the third question we posed at the beginning of this section: how can the Planck constant, $\hbar$ we used in the definition of the wave function (1) be interpreted in a macro-scopic context? Khrennikov [14–16] and Choustova [4] give interesting ideas in this regard. They propose the Planck constant could be compared to a price scaling parameter and it could possibly be made time dependent. In [8] we propose that the macro-scopic version of $\hbar$, which we denote there as $e_\$$, could be some rate of interest reflecting the inherent level of uncertainty in the economy.

4 A Change of the Information Wave Function and the Inducement of Arbitrage

The existence of arbitrage is for a large part based upon the existence of information. Hence, changes in information will alter arbitrage opportunities. We can thus imagine a benchmark situation where we start out under no arbitrage with a particular state of information, reflected by a particular functional form of the information wave function. Our goal, in the Proposition 4.1. below, is then to show that if the state of information changes (this can for instance be reflected by a change in the functional form of the wave function) arbitrage can occur. Before we consider our proposition, let us consider again Theorem 2.1. We recall that if there exists a K dimensional state price vector $\overrightarrow{\Phi} = (\Phi_1, \Phi_2, \Phi_3, \ldots, \Phi_K)$ which solved the system of equations (of Theorem 2.1) there will be no arbitrage (and vice versa). We defined an associated probability vector $\overrightarrow{\Phi_{\text{prob}}} = (\frac{\Phi_1}{\Phi_0}, \frac{\Phi_2}{\Phi_0}, \frac{\Phi_3}{\Phi_0}, \ldots, \frac{\Phi_K}{\Phi_0})$, where each coordinate is a probability. We have also that $\Phi_0 = \exp(-rT)$ where r is the risk-free rate of return and T is time. We must stress that we assume that the probability values making up the probability vector $\overrightarrow{\Phi_{\text{prob}}}$ (from Theorem 2.1) can be drawn from $\|\Psi\|^2 = \int_{\mathbb{R}} |\Psi(q)|^2 dM(q)$.

Proposition 4.1 *Let there exist an N-dimensional asset price vector $\overrightarrow{p_0}$ and a K-dimensional state price vector $\overrightarrow{\Phi}$. Let there exist a K-dimensional probability vector $\overrightarrow{\Phi_{\text{prob}}} = (\frac{\Phi_1}{\Phi_0}, \frac{\Phi_2}{\Phi_0}, \frac{\Phi_3}{\Phi_0}, \ldots, \frac{\Phi_K}{\Phi_0})$. Let $N = K$ and let $\Phi_0 = \exp(-rT)$ be fixed. Let there be an information wave function $\Psi(q)$ and a measure M on $\mathbb{R}$. Let each of the probabilities in $\overrightarrow{\Phi_{\text{prob}}}$ be drawn from $\|\Psi\|^2 = \int_{\mathbb{R}} |\Psi(q)|^2 dM(q)$ for each of a respective set of lower and upper bound values of the integral. Let the state prices which are in $\overrightarrow{\Phi_{\text{prob}}}$ guarantee no arbitrage. Consider now an information wave function $\Upsilon(q)$ which has a different functional form from $\Psi(q)$ in the following way*: (i) *we assume that $\Upsilon(q)$ can not be the dual wave function of $\Psi(q)$;*[1] (ii) *for the same measure M on $\mathbb{R}$ and for the same respective set of lower and upper bound values of the integral we used for $\int_{\mathbb{R}} |\Psi(q)|^2 dM(q)$, we write $\int_{\mathbb{R}} |\Upsilon(q)|^2 dM(q)$ such that the functions $|\Upsilon(q)|^2$ and $|\Psi(q)|^2$ can be allowed to intersect on different intervals of their domain* (*of course the functions may not intersect at all*) *but under the constraint that at least one probability drawn from $\|\Upsilon\|^2 = \int_{\mathbb{R}} |\Upsilon(q)|^2 dM(q)$ must be different from the probabilities drawn from $\int_{\mathbb{R}} |\Psi(q)|^2 dM(q)$. Under those conditions will the change in the information wave function from $\Psi(q)$ to $\Upsilon(q)$ trigger arbitrage.*

Proof Since $N = K$ there will be a unique state price vector $\overrightarrow{\Phi} = (\Phi_1, \Phi_2, \ldots, \Phi_K)$ solving the system of equations set out by the non-arbitrage theorem. Hence, the probabilities in $\overrightarrow{\Phi_{\text{prob}}} = (\frac{\Phi_1}{\Phi_0}, \frac{\Phi_2}{\Phi_0}, \frac{\Phi_3}{\Phi_0}, \ldots, \frac{\Phi_K}{\Phi_0})$, where $\Phi_0 = \exp(-rT)$ is fixed are also unique. Those probabilities are drawn from $\|\Psi\|^2 = \int_{\mathbb{R}} |\Psi(q)|^2 dM(q)$, for each of the respective set of lower and upper bound values of the integral. Now let us consider an information wave function of a different functional form, $\Upsilon(q)$. This information wave function can not be the dual of the information wave function $\Psi(q)$. As pointed out by one of the referees of the paper, the space of the information wave functions (with the Hermitian inner product) is a Hilbert space. As the referee points out "Hence (by the Riesz' Lemma) the dual and bidual spaces are conjugate isomorphic, respectively isometrically isomorphic to the Hilbert space." In that case the result will not hold. Keeping this in mind, we do allow both $|\Psi(q)|^2$ and $|\Upsilon(q)|^2$ to overlap on different intervals (of course the functions may not overlap at all) in their domain. However, the overlap is permissible up to the point where we require, for the same respective set of lower and upper bound values of the integrals, that at least one probability value generated by $\|\Upsilon\|^2 = \int_{\mathbb{R}} |\Upsilon(q)|^2 dM(q)$ must be different from any of the probability values generated by $\|\Psi\|^2 = \int_{\mathbb{R}} |\Psi(q)|^2 dM(q)$. Without this restriction, the two different functions could possibly overlap on intervals of their domain in such a way to generate (for the same lower and upper bound values of the integrals) exactly the same probabilities. We are now sure that the different functional form of the information wave function will induce at least one different probability. Therefore, the emerging probability vector $\overrightarrow{\Phi^{***}_{\text{prob}}} = (\frac{\Phi_1^{***}}{\Phi_0}, \frac{\Phi_2^{***}}{\Phi_0}, \frac{\Phi_3^{***}}{\Phi_0}, \ldots, \frac{\Phi_K^{***}}{\Phi_0})$, which has now probabilities uniquely drawn from $\|\Upsilon\|^2 = \int_{\mathbb{R}} |\Upsilon(q)|^2 dM(q)$ will contain at least one state price in $\overrightarrow{\Phi^{***}} = (\Phi_1^{***}, \Phi_2^{***}, \ldots, \Phi_K^{***})$ such that $\overrightarrow{\Phi^{***}} \neq \overrightarrow{\Phi}$. Hence, there must be arbitrage. □

We make the following three remarks.

[1]Thanks to one of the referees for pointing out this important restriction.

1. It is feasible to trigger arbitrage by only changing the upper and lower bound values of the integral and keeping the functional form of the wave function unchanged. In this sense can q be interpreted as the price of information. Since arbitrage is dependent on information, a change in the price of information could trigger arbitrage. As we have already indicated before (please see Sect. 3), we can think of the price of information as the price of proprietary information for instance. A change in the functional form of the information wave function would then indicate a change in the information about the price of information.
2. If we allow a change can occur in the risk free interest rate, r so that Φ_0 is not fixed anymore, then the change in the functional form of the information wave function from $\Psi(q)$ to $\Upsilon(q)$ could induce a change from the risk free rate of return r to another rate of return, say R, which is non-risk free. The difference between $R - r$ could be denoted as a risk premium (or a risk discount) if respectively the difference is positive or negative. Alternatively, we could also change T, although that would have little economic meaning. Clearly, in both cases the state prices will also need to change since the probabilities need to continue to add up to unity.
3. Our proposition will not necessarily hold for the case where $N > K$ since in this case we may obtain more than one set of state price vectors guaranteeing non-arbitrage. Hence, for a different functional form $\Upsilon(q)$ it could still be possible we find state prices guaranteeing non-arbitrage.

5 How Can We Connect Bohmian Mechanics to the Non-Arbitrage Theorem?

Using the conditions contained in the above Proposition 4.1 we observe, that in the case when $N = K$, a change in the information wave function from $\Psi(q)$ to $\Upsilon(q)$, will change the state price vector $\overrightarrow{\Phi} = (\Phi_1, \Phi_2, \ldots, \Phi_K)$ to $\overrightarrow{\Phi^{***}} = (\Phi_1^{***}, \Phi_2^{***}, \ldots, \Phi_K^{***})$ such that $\overrightarrow{\Phi^{***}} \neq \overrightarrow{\Phi}$. We need at least one state price in $\overrightarrow{\Phi^{***}}$ which is different from the state prices in $\overrightarrow{\Phi}$. From Theorem 2.1, we know that the first state price, Φ_1, multiplies all the prices contained in the N dimensional asset price vector corresponding to state 1. We continue doing this for all K states. Neftci [17] provides for an interesting interpretation of the state prices and likens them to prices used in an insurance policy. As an example, using the notation of Theorem 2.1, consider the price of asset 2 at time 0, which we denoted as p_0^2. An investor could be willing to pay Φ_1 units for an 'insurance policy' that offers D_{21} units of currency if state 1 (at time 1) is to occur (but the insurance pays nothing if any other state than state 1 occurs). The investor could be willing to pay Φ_2 units plus Φ_1 units for an 'insurance policy' that offers D_{22} units of currency if state 2 (at time 1) is to occur and D_{21} units of currency if state 1 (at time 1) is to occur (but the insurance pays nothing if any other state than states 1 and 2 occur). If the investor wants to insure that he gets a payoff no matter what state occurs, then he will be willing to pay an 'insurance policy' of $\Phi_1 + \Phi_2 + \cdots + \Phi_K$.

In order to connect our theory with Bohmian mechanics we would need to introduce the idea of a continuous state space. We would then, in light of the above, have a continuum of 'insurance policy' prices. In a Bohmian mechanics environment, we would have a continuum of information prices, q and the information wave functions would change following the Schrödinger partial differential equation. So the information about the information prices changes as well as the information prices themselves. Those changes will then affect the 'insurance policy' prices.

So we could in physics terms, imagine there exists a smooth information price trajectory which is traced out by the Newton-Bohm equation:

$$m\frac{d^2q(t)}{dt^2} = -\frac{\partial V(q)}{\partial q} - \frac{\partial Q(q)}{\partial q}, \tag{3}$$

subject to the initial conditions that $q(t=0) = q_0$ and $q'(t=0) = q_0^{'}$, where q_0 is the information price at $t=0$ and $q_0^{'}$ is momentum at $t=0$. We note that m is mass and $\frac{d^2q(t)}{dt^2}$ is acceleration; $-\frac{\partial V(q)}{\partial q}$ is the partial derivative of the real potential towards the information price and $-\frac{\partial Q(q)}{\partial q}$ is the partial derivative of the quantum potential towards the information price. We note that m, the real potential V and the quantum potential Q, have already been interpreted economically in the work by Khrennikov [14–16], Choustova [4] and Haven [8–10]. In Bohmian mechanics, since the quantum potential Q, depends on the wave function (via the amplitude function of the wave function) we can say the wave function steers the particle. Thus, in the context we have now described, the *information* about the price of information (i.e. the information wave function) steers the information prices. Moreover, it seems reasonable to claim that the smooth information price trajectory on the prices of information would also trigger a trajectory of 'insurance policy' prices.

In summary, we have obtained, via the use of the information wave function, a natural device by which we can induce either arbitrage or form a risk premium (or risk discount). Those two financial phenomena are essential in asset pricing and hence we can begin to see the importance of the information wave function in an asset pricing context.

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